Value capture in hierarchically organized value chains


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Abstract: We study how the structure of negotiations in a value chain affects the distribution of value among its members. To this end, we generalize the Shapley value and the core to hierarchical bargaining situations. While the core yields no concrete predictions, the Shapley value analysis suggests that positions most conducive to value capture are those that allow to realize large complementarity gains. If the game exhibits “super-complementarity,” then it is advantageous if a firm’s negotiation partners are grouped into clusters. Using examples from the aircraft and white goods industries, we assess whether the firms’ actions are consistent with model predictions.

Keywords: value capture, negotiations, Shapley value, industry architecture, modularity

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1. Introduction

The past decade has seen significant structural shifts in the value chain of large commercial aircraft. Airbus and Boeing, traditionally integrators of a large number of aircraft components, have handed over responsibility for large sections of the aircraft to select suppliers for the most recent programs of the A350 and the B787 Dreamliner. These so-called mega suppliers not only design and integrate the awarded sections but also manage the value chain for the respective system. Concomitantly, firms that once dealt directly with the original equipment manufacturer (OEM) have been moved to a lower tier where they now negotiate with and supply to one of the mega suppliers. For example, B/E Aerospace, which supplies oxygen systems and used to deal with Airbus directly for earlier programs, now supplies components for the A350 program to and conducts all related price negotiations with mega supplier Spirit AeroSystems.

Arguably, such changes to the value chain should affect the distribution of value among the participating firms; yet, the extant literature on value capture largely focuses on a firm’s replaceability. According to this logic, a favorable position with respect to resources and market structure may put a firm in a “bottleneck” position in an industry and, more specifically, in a value chain (Baldwin, 2015; Iansiti and Levien, 2004; Jacobides and MacDuffie, 2013; Jacobides et al., 2006; Morris and Ferguson, 1993; Pisano and Teece, 2007). While existing studies have greatly enhanced our understanding of how industry architecture affects value capture through the intensity of competition in the various segments of a value chain, more fundamental variations in the architecture of value chains are ubiquitous, as illustrated by the aircraft example. Such changes to value chains, implying that its members are rearranged, are not considered in the existing literature. In fact, the meaning of “architecture” as describing the structure of the value chain and the linkages among its constituent firms is, so far, largely unexplored. The study by Erat et al.
(2013) is a notable exception; however, it focuses on competitive differentiation and the benefits of outsourcing the integration function rather than on effects of the structure of negotiations.

The present study—based on and extending the work by Hoffmann (2015)—addresses this gap. Focusing on the value chain as the unit of analysis, we argue that the division of value between its member firms takes place in a hierarchy of negotiations. Typically, the manufacturer of a final good will negotiate with its tier-1 suppliers, which, in turn, will negotiate with their respective suppliers. We refer to the *bargaining structure* of a value chain to describe which of its members negotiate among each other in the various stages and branches of the value chain, and how these individual negotiation processes are linked.

A simple example of three firms shows how bargaining structure affects value distribution. We assume that costs are zero and that all firms are essential, in the sense that each firm’s absence would reduce the overall value captured by the remaining firms to zero. There are two possible bargaining structures, non-hierarchical and hierarchical. In the non-hierarchical, or *linear*, structure, all three firms bargain jointly on the same level. The most plausible prediction of payoffs, in line with the Shapley value (Shapley, 1953), is that each firm obtains one-third of the total value, for reasons of symmetry. A hierarchical structure, in contrast, is given if one firm bargains on the top level with a representative of the other two firms (that together form a “cluster”), which subsequently negotiate to split among each other the value captured by their representative. The two top-level negotiators are symmetric in that both are essential and have zero cost; therefore, they should split the available value evenly. This outcome may appear counterintuitive, since one of the negotiators represents two players. However, the two-firm representative can threaten only once to withdraw from the negotiation, just as can its counterpart. Furthermore, the single firm may point to the possibility that it could split its position in two, referring, for example, to two process steps or two components of its input.¹ Thus, what one
perceives as an intuitive outcome of the top-level negotiation depends on the point of reference. This may be either a situation with three firms, two of which are represented by one negotiator, or a situation with two negotiators, one of which has an internal structure. With the latter reference point, an even split on the top level is intuitive. Research on cognitive biases in decisions on how to allocate budget between divisions of a firm suggests that the latter reference point is indeed the salient one (Bardolet et al., 2011). In the second-level negotiation, the two firms would again arrive at an equal split given that both are essential, each obtaining one quarter. Thus, in this example, a hierarchical bargaining structure dramatically favors the single firm.

Using cooperative game theory, we analyze how the bargaining structure of the value chain for a specific product affects the distribution of value among the contributing firms. We generalize the Shapley value (Shapley, 1953) and the core (Gillies, 1953; Shapley, 1952) by introducing the concepts of the hierarchical Shapley value and the hierarchical core. We then use these concepts to study the effects of bargaining structure on the value split. Our research thus also contributes to the literature on value capture theory (see the review by Gans and Ryall, 2017).

Our main findings are the following. The hierarchical core places the same bounds on the amount of value that a cluster can capture in a hierarchical bargaining structure as the core does in a linear bargaining structure. The hierarchical Shapley value, in contrast, makes predictions that differ from those of the standard Shapley value. To state them, we introduce the concepts of complementarity gains—the increment in value that players create by acting jointly over the sum of what they create individually—and of super-complementarity, which, intuitively, means that larger complementarity gains are realized toward the final levels of the value chain. With super-complementarity, a participant in the top-level negotiation benefits if other participants are merged into clusters; in particular, a bargaining structure consisting of one firm and one cluster is advantageous to the single firm compared with linear bargaining, a two-cluster hierarchical
structure in a symmetric game is advantageous to the smaller cluster, and a merger of two firms or clusters in the top-level negotiation is advantageous to the other negotiators.

Given that the bargaining structure affects the distribution of value, each firm has an incentive to shape the value chain in such a way as to maximize its own value capture—though few will be in a position to accomplish this. In a qualitative empirical study of cases from the commercial aircraft and white goods (major appliance) industries, we show that bargaining structures are indeed malleable and, to some extent, under the control of the central firm. We furthermore employ these examples to illustrate the predictions derived from our model. Following Baldwin and Clark (2000), Colfer and Baldwin (2010), and Henderson and Clark (1990) we suggest that firms can leverage a modular product architecture to shape industry architecture, the respective value chain architecture, and, thus, the bargaining structure.

2. Related Literature

2.1. Value Capture

We define a value chain following Sturgeon (2001) as the collectivity of all firms that contribute to a particular instance of value creation through a specific division of labor. For the purpose of our analysis, we focus on those layers of the value chain that provide inputs close to or specific to the final product. Value is split among firms in a value chain through bargaining (Brandenburger and Stuart, 2007) against the background of resource ownership (Barney, 1986; Daft, 1983) and market structure (Bain, 1956; Porter, 1980). Since we focus on the distribution of value within a value chain, we take the value captured by the value chain as a whole as given. We thus follow prior research (Bowman and Ambrosini, 2000; Dedrick et al., 2010; Lepak et al., 2007) in defining value—more specifically, “value captured”—as the difference between the price buyers pay for a certain good and the costs of producing it.
Building on the seminal work of Teece (1986), the literature on profiting from innovation emphasizes the appropriability regime and control over complementary assets as the key drivers of value capture. Both may allow a firm to create a “bottleneck” (Baldwin, 2015; Iansiti and Levien, 2004; Jacobides and MacDuffie, 2013; Jacobides et al., 2006; Morris and Ferguson, 1993; Pisano and Teece, 2007) or, in other words, to become essential. More generally, the irreplaceability and inimitability that being a bottleneck implies may be due to various isolating mechanisms (Rumelt, 1984), in particular causal ambiguity and legal property rights. We refer to the approach to optimizing a firm’s value capture by becoming a bottleneck as the bottleneck strategy.

Beyond a firm’s replaceability, further determinants of bargaining power are, switching costs when deploying its resources for other purposes, time pressure, access to relevant information, and the sequence of the bargaining process (Bennett, 2013; Buvik and Reve, 2002; Dedrick et al., 2010; Porter, 1980). Also an “IP-modular” product structure may improve a firm’s bargaining position (Baldwin and Henkel, 2015; Henkel et al., 2013). What is lacking in the literature, and what we address in this paper, is the role that the value chain architecture plays in value capture.

2.2. Value Chain Architecture

Distinct product solutions designed to satisfy the same customer need might have different value chain architectures. In particular, when a new market emerges, firms approach market needs and process difficulties differently, and each product design may come with its own organization of the value chain (Clark, 1985). However, when a dominant product design emerges (Abernathy and Utterback, 1978), most firms will adopt the corresponding value chain organization in order to reduce transaction costs. Thus, one or a small number of stable “industry architectures,” consisting of several value chains for core and complementing products, emerge in the formative years of an industry (Gawer and Cusumano, 2002; Iansiti and Levien, 2004).
Firms may try to adapt an industry architecture so that it acts to their advantage through fostering competition in other segments of the value chain, for instance by establishing open interfaces, and through reducing competition in their own segment, for example, with the help of legal protection mechanisms such as patents (Eisenmann et al., 2009; Jacobides et al., 2006). Such changes can have important implications for a firm’s value capture, but are limited to individual segments and leave the overall value chain architecture intact.

While industry and value chain architectures can display significant inertia (Pisano and Teece, 2007), architectures can change over time, triggered, for example, by technological and regulatory changes or demand shifts (Jacobides et al., 2006), or by the OEM’s desire to reduce supplier complexity. Players in a position to shape the value chain architecture may be innovators (Jacobides et al., 2006), entrepreneurs (Santos and Eisenhardt, 2009), or other key firms leveraging their position or assets (Ferraro and Gurses, 2009; Iansiti and Levien, 2004). Specifically, the product architecture that an innovator chooses, in particular its modularity, affects the division of labor within an industry and, hence, value chain architecture (Baldwin and Clark, 1997; Langlois, 2003; Langlois and Robertson, 1992; Sanchez and Mahoney, 1996; Sturgeon, 2002), and an OEM may force changes to the value chain architecture through its sourcing decisions (e.g., Jacobides et al., 2015; Novak and Wernerfelt, 2012). To the extent that a change in the value chain architecture entails a corresponding change in the bargaining structure—a question we address in Section 3.2—firms may try to shape their value chain architecture to optimize value capture.

2.3. Game Theoretic Perspective

Cooperative game theory, introduced to the management literature by Brandenburger and Stuart (1996) to develop the concept of added value, is suited to analyze situations in which binding contracts among players can be signed and adhered to (Aumann and Shapley, 1994). The literature on value capture theory, recently reviewed by Gans and Ryall (2017), typically employs the core
solution concept to analyze variations in competitive intensity along the value chain and their drivers. Its goal is not to predict precise outcomes of strategic interactions; rather, as Gans and Ryall (2017, p. 22, italics in original) put it, “the value capture model suggests that competition is properly construed as placing *bounds* on the amount of value an agent may capture without fully determining it.”

Specifically, MacDonald and Ryall (2004) study how competition and replaceability affect value appropriation, a question that Montez et al. (2017) extend to include competition for the focal firm. Chatain and Zemsky (2011) analyze the effect of “frictions”—incomplete linkages in the industry value chain due to search and switching costs that prevent firms from forming coalitions—on value creation and value capture. Similar incomplete linkages appear in our model because firms in different segments of the value chain are separated from each other. Finally, Ryall and Sorenson’s (2007) analysis of the conditions under which brokers hold a competitive advantage exhibits parallels to our study of value chain architecture.

The most prominent solution concepts in cooperative game theory are the Shapley value (Shapley, 1953) and the core (Gillies, 1953; Shapley, 1952). While the core identifies a set of value distributions that no group of players can unilaterally improve upon, the Shapley value provides a unique distribution. In the meta-analysis by Michener et al. (1983), the Shapley value consistently shows higher predictive accuracy than other solution concepts. The fields of economics and political science extensively use the Shapley value. Yet, so far, only a few applications in management studies exist, notably by Granot and Sošić (2005), Hendrikse (2011), Layne-Farrar et al. (2007), and Kattuman et al. (2011). As we will show, the Shapley value and its generalization to hierarchical bargaining produce concrete and economically plausible results in our context, while the core yields no concrete predictions.
3. Bargaining Structure

3.1. Hierarchy in Negotiations: Defining Bargaining Structure

When several parties have to split a given value, one or more negotiations may take place. For example, the parties may be divided into two groups and the distribution of value between them negotiated by representatives from each group. The value obtained by each group is subsequently split in a further negotiation between its members. We define the *bargaining structure* of the value distribution process as a division of the set of players into disjoint subsets, each of which may, in turn, consist of disjoint subsets, and so on until all lowest-level subsets contain only one party.²

With the exception of simple cases and vertically integrated value chains, such hierarchical negotiations do occur in most real-life value chains. Bargaining will generally be a hierarchical, multi-stage process, particularly with large numbers of participating firms; one reason for this is to reduce transaction costs (Williamson, 1979). The manufacturer of the final good will most commonly negotiate with its tier-1 suppliers (though there are exceptions); these firms, in turn, will negotiate with their own suppliers (which are tier-2 suppliers from the perspective of the OEM). For complex products, this chain of negotiations may continue for several more stages. In general, it will have no clearly defined ending point since even suppliers of raw materials have their own suppliers of machinery. For the purpose of our argument, we focus on those parts of the bargaining structure that are close to the final product. Since adding an additional level does not affect the distribution of value in the levels above, the choice of which levels to include is not critical to our analysis.

3.2. Determinants and Effects of Bargaining Structure

To our knowledge, while scholars have focused on how firms can influence their own and others’ replaceability in a given bargaining structure, the bargaining structure itself has received no attention as a potential lever for enhancing value capture. As bargaining structure determines the
participants of each negotiation, and as value capture in a specific negotiation depends on each party’s bargaining power relative to that of the others, we suggest that the resulting value distribution depends on the bargaining structure. Thus, we need to ask what, in turn, determines bargaining structure. We argue that value chain architecture and individual participants’ power are the key determinants, and address them in turn.

As the architecture of a value chain circumscribes the division of labor and the roles of firms that participate in producing the product (Jacobides et al., 2006), it also outlines the transactions of goods and provision of services from upstream to downstream firms. Transactions, in turn, are governed by contracts; hence, the value chain architecture gives rise to a transaction-related contract structure. The terms of transaction-related contracts include the price, thus directly determining the distribution of value. As transaction-related contracts are commonly subject to negotiations (Nagarajan and Sošić, 2008), their structure will, in most cases, be congruent to the bargaining structure, thus making the value chain architecture a key determinant of the latter. Consequently, firms in a position to control value chain architecture (see Section 2.2) can use their power to shape bargaining structure to their advantage. For example, a final goods manufacturer may buy its inputs directly from a large number of suppliers, or may alternatively choose some of them as tier-1 suppliers that provide integrated subsystems and that are, in turn, each supplied by—and negotiating with—a subset of the other suppliers (which thus become tier-2 suppliers to the manufacturer). Airbus’ moving B/E Aerospace to a tier-2 position is a case in point. Since a bargaining structure defines a hierarchy of negotiations, we refer to a firm’s approach to optimizing its value capture through creating a favorable bargaining structure as the hierarchy strategy.

In addition, based on their assets or their position in the industry some firms might have the power to shape the bargaining structure directly. For example, automotive OEMs may opt to
negotiate directly with a tier-2 supplier to realize quantity discounts, shutting out the respective tier-1 supplier (MacDuffie and Helper, 2007). In such a case, the OEM does not change the role of the tier-1 supplier in the value chain architecture—the tier-1 supplier remains responsible for the subsystem and its integration—but it does modify the bargaining structure.

4. The Model

4.1. Cooperative Game Theory, the Shapley Value, and the Core

The distribution of value within a value chain typically involves several or even many interactions between participating firms. Since non-cooperative game theory would require the specification of “protocols” for each interaction (Brandenburger and Stuart, 1996), we model the bargaining over value as a cooperative game. Furthermore, cooperative game theory assumes that binding contracts can be written, which is naturally the case for firms collaborating within a value chain.

Our analysis focuses on value distribution under a given bargaining structure. We assume as given, the set of firms among which the total value is split, the cost of production, and the value that the value chain captures as a whole. We do not explicitly consider suppliers of unspecific inputs, buyers of the final products, and competitors of the firms in the value chain. These actors are “outside the game” (Brandenburger and Stuart, 1996: 11), in that the prices they charge or pay are given. Equally, the options they provide the players in the value chain—for example, to replace a supplier with an outside firm—are fixed. These outside options and prices influence the negotiation position of each player within the game, and are reflected in the value that each group of players, particularly each player individually, can appropriate.

We base our model on the most prominent solution concepts for cooperative games, i.e., the Shapley value (SV) (Shapley, 1953) and the core (Gillies, 1953; Shapley, 1952). The SV has a number of desirable and plausible properties and shows a relatively good predictive accuracy (Michener et al., 1983; Michener et al., 1987). The core, in contrast, has the advantage of requiring
fewer assumptions and thus introducing less specificity into the analysis (Brandenburger and Stuart, 1996). As we will see, however, the core is mostly agnostic when comparing bargaining structures.

4.2. Non-hierarchical Value Chains

In a non-hierarchical situation we describe the value split between $n$ firms by a cooperative game $G$ characterized by the player set, $M = \{m_j\}_{j=1,...,n}, n \in \mathbb{N}$, and the characteristic function, $v: 2^M \rightarrow \mathbb{R}$. We assume that $v$ is convex, i.e., $v(J \cup K) + v(J \cap K) \geq v(J) + v(K)$ for all $J, K \subseteq M$. We call a firm essential if any coalition not comprising this firm would, on its own, capture a value of zero. We introduce, for $J \cap K = \emptyset$, the notion of complementarity gains:

$$\Delta_{J,K} := v(J \cup K) - v(J) - v(K)$$

As solution concepts, we employ the SV and the core. The SV of player $m_j \in M$ is defined as this player’s average marginal contribution to all possible sequences of players:

$$\phi_{m_j}(v) = \sum_{S \subseteq M \setminus \{m_j\}} \frac{|S|!(n-|S|-1)!}{n!} \left( v(S \cup \{m_j\}) - v(S) \right)$$

$$= v(\{m_j\}) + \sum_{S \subseteq M \setminus \{m_j\}} \frac{|S|!(n-|S|-1)!}{n!} \Delta_{S\cup\{m_j\}}.$$  (2)

The core comprises all allocations $x \in \mathbb{R}^{|M|}$ that grant each coalition $J$ at least the value that it can appropriate stand-alone. Since the game is convex, the core is non-empty (Shapley, 1971). Introducing the notation $x(J) := \sum_{j \in J} x_j$ for $J \subseteq M$, we have:

$$C(M,v) = \{ x \in \mathbb{R}^{|M|} | x(J) \geq v(J) \ \forall \ J \subseteq M \ \land \ x(M) = v(M) \}. \quad (4)$$
4.3. Hierarchical Cooperative Games

4.3.1. Hierarchical bargaining structure

We now consider a hierarchical bargaining structure with two levels. \( M \) is divided into \( k \in \mathbb{N} \) subsets, which we refer to as clusters. The representatives of these clusters bargain in a top-level, or level-1, (L1) negotiation. Subsequently, the members of each cluster bargain among each other in level-2 (L2) negotiations to split their respective group’s bounty. For a value chain, an OEM and its tier-1 suppliers negotiate in an L1 negotiation; subsequently, each tier-1 supplier and its own suppliers (tier-2 suppliers for the OEM) negotiate in an L2 negotiation. Each tier-1 supplier and the tier-2 suppliers supplying it constitute a cluster; the OEM constitutes a cluster by itself.

Formally, we build on the definition of coalition structures by Aumann and Drèze (1974) to model hierarchical bargaining structures. A hierarchical bargaining structure is a partition \( \mathfrak{B} = \{M_i\}_{i=1}^k \) of the set \( M \) of firms into \( k \) clusters \( M_i \) such that \( M = \bigcup_{i=1}^k M_i \) (the elements of \( \mathfrak{B} \) cover \( M \)), and \( M_i \cap M_j = \emptyset \ \forall M_i, M_j \in \mathfrak{B}: i \neq j \) (the elements of \( \mathfrak{B} \) are pairwise disjoint).

4.3.2. Value distribution between clusters

Based on \( G \) and \( \mathfrak{B} \) we define a hierarchical cooperative game, \( G_\mathfrak{B} \), consisting of \( 1 + k \) standard cooperative games. The top-level game, \( L1 \) game or quotient game, is characterized by the player set \( \mathfrak{B} \) and the characteristic function \( \hat{v} \), and describes the value distribution among the clusters. We follow Owen (1977) and Pulido and Sánchez-Soriano (2009) in defining \( \hat{v} \), which derives from \( v \) in a natural fashion, as follows:

\[
\hat{v}: 2^{\mathfrak{B}} \to \mathbb{R}, \quad \hat{v}(J) := v \left( \bigcup_{M_i \in J} M_i \right) \quad \forall \ J \subseteq \mathfrak{B}. \tag{5}
\]

As solution concepts for the hierarchical game we introduce the hierarchical Shapley value (HSV) and the hierarchical core (HC). For the L1 game played between the clusters, they are
identical to the Owen value (Owen, 1977) and the co-aliational core (Pulido and Sánchez-Soriano, 2009), respectively. The solutions to the L1 game derived from these concepts are natural. The L1 HSV for a cluster $M_i \in \mathfrak{B}$, $\phi_{M_i}(\mathfrak{B}, \hat{v})$, and the L1 HC, $HC_{L1}(\mathfrak{B}, \hat{v})$ are given by, respectively (with $y(S) := \sum_{M_i \in S} y_i$ for $S \subseteq \mathfrak{B}$):

$$\phi_{M_i}(\mathfrak{B}, \hat{v}) := \sum_{S \subseteq \mathfrak{B} \setminus \{M_i\}} \frac{|S|! (k - |S| - 1)!}{k!} \left( \hat{v}(S \cup M_i) - \hat{v}(S) \right); \quad (6)$$

$$HC_{L1}(\mathfrak{B}, \hat{v}) := \{ y \in \mathbb{R}^{[\mathfrak{B}]} | y(S) \geq \hat{v}(S) \forall S \subset \mathfrak{B} \land y(\mathfrak{B}) = \hat{v}(M) \}. \quad (7)$$

Only the marginal contribution of a (complete) cluster to each coalition of other clusters matters; the internal composition of the clusters is irrelevant. This assumption is plausible since each L1 negotiator can only threaten once to withdraw from the negotiation. Furthermore, single firms on the top level can argue that they, too, could split into different parties (e.g., business units), and thus dismiss the argument of other L1 negotiators that those represent more than one firm. Relatedly, in firm-internal budget allocation decisions between divisions, Bardolet et al. (2011, p. 1468) have identified a “partition dependence.” Their empirical results show that a top manager directs his or her attention toward the top level of the hierarchy, the composition of which thus has a significant influence on the budget allocation. We argue that the grouping of firms into clusters in L1 negotiations has an analogous effect.

4.3.3. Value distribution within clusters

The remaining $k$ games describe the value distribution on L2, within each of the $i = 1 \ldots k$ clusters with player sets $M_i$. We denote the L2 characteristic function on $M_i$ by $\tilde{v}_{M_i}: 2^{M_i} \rightarrow \mathbb{R}$. It needs to satisfy two conditions: first, $\tilde{v}_{M_i}(\emptyset) = 0$; and second, $\tilde{v}_{M_i}(M_i)$ must equal the allocation that $M_i$ has received in the L1 negotiation (“efficiency”). For comparability, it would be desirable to use the same L2 characteristic function for the HSV and the HC. However, since the L2 characteristic
function depends on the outcome of the L1 game, this is impossible without mixing the concepts.\footnote{We thus define, for $J \subseteq M_i$, the L2 characteristic functions for HSV and the HC as follows:}

\[
\tilde{v}^{\text{HSV}}_{M_i}(J) := \frac{v(J \cup (M \setminus M_i)) - v(M \setminus M_i)}{v(M) - v(M \setminus M_i)} \phi_{M_i}(\mathcal{B}, \tilde{v}) ;
\]

\[
\tilde{v}^{\text{HC}}_{M_i}(J) := \max \{v(J); v(J \cup (M \setminus M_i)) - y(M \setminus M_i)\} .
\]

Both definitions reflect the assumed dependencies between clusters, that is, the fact that other clusters $(M \setminus M_i)$ play a role in determining the value created by a coalition $J \subseteq M_i$. Specifically, the terms $v(J \cup (M \setminus M_i)) - v(M \setminus M_i)$ and $v(J \cup (M \setminus M_i)) - y(M \setminus M_i)$ capture our assumption that the players within a cluster assume all other clusters $(M \setminus M_i)$ to be complete and in place, and a coalition’s incremental contribution over $v(M \setminus M_i)$ matters for its characteristic function. The linkage to the L1 game is established, for the HSV (8), through multiplication with $\phi_{M_i}(\mathcal{B}, \tilde{v})$. The ratio in (8) can be interpreted as the importance of coalition $J \subseteq M_i$ to the other clusters $(M \setminus M_i)$ relative to the importance of the complete cluster $M_i$ to the other clusters. Since the ratio equals unity for $J = M_i$, the definition ensures efficiency, i.e., that the grand coalition $M_i$ captures the entire value allocated to this cluster on L1. While alternative definitions of the L2 HSV of $J \subseteq M_i$ may be possible—e.g., proportional to $\left( v(M_i) - v(M_i \setminus J) \right)$—the one we propose remains as closely as possible to the characteristic function of the underlying game. For the HC (9), the linkage to the L1 game is reflected in the superscript, which indicates that $\tilde{v}^{\text{HC}}_{M_i}$ depends on the L1 core allocation, described by the vector $y$. The fact that $\tilde{v}^{\text{HC}}_{M_i}(M_i) = v(M_i \cup (M \setminus M_i)) - y(M \setminus M_i) \equiv y(M_i)$ ensures efficiency (the first equality holds because $v(M_i) \leq y(M_i)$). Note that for the HC, $J \subseteq M_i$ may create value alone or jointly with $M \setminus M_i$, but not with a subset of the constituent clusters.\footnote{The assumption that for negotiations within a cluster all other clusters are complete and in place distinguishes the HSV and the HC from the Owen value and the coalitional core, respectively. In the real world, it will be fulfilled if negotiators have limited transparency about the}
contributions of other clusters and their constituent firms, a plausible assumption similar to the feature of “information hiding” in modular systems (Baldwin and Clark, 2000, p. 73). They might also consider marginal contributions they would make to other clusters to be too far off to use them as justification for demands they make in their own L2 negotiation.

Based on the L2 characteristic functions (8) and (9), the L2 HSV and HC obtain in the standard fashion. For simplicity, we use \( \phi_j^{HSV}(v) \) to refer to the value capture \( \phi_{m_j}^{HSV}(\tilde{v}_{M_i}, \mathfrak{B}) \) of a firm \( m_j \) in a hierarchical bargaining structure \( \mathfrak{B} \). Formally, the HC is defined as follows (where \( y \in \mathbb{R}^{\mathfrak{B}} \) is given by \( y_i = x(M_i) \)):

\[
HC(M, \mathfrak{B}, v) := \{ x \in \mathbb{R}^n \mid y \in HC_{L_1}(\mathfrak{B}, \hat{v}) \land x(S) \geq \tilde{v}_{M_i}^Y(S) \forall M_i \in \mathfrak{B}, \forall S \subseteq M_i \}. \tag{10}
\]

To illustrate an L2 negotiation, consider the case of a tier-1 supplier negotiating with its own suppliers (tier-2 suppliers for the OEM). We note that such negotiations will typically take place in bilateral interactions, while the HSV and the HC treat all firms in a given cluster symmetrically. However, as discussed above for the non-hierarchical case, they are nonetheless suitable as solution concepts since their symmetry is not meant to reflect the actual organization of negotiations. A seeming contradiction may lie in the dual role of the tier-1 supplier as a representative of its cluster in the L1 negotiation, and as an opponent of the other cluster members in the L2 negotiation. These roles are fully consistent: no matter how tough the tier-1 firm negotiates in its L2 negotiation, it always has an interest in maximizing its outcome in the L1 round. Finally, the tier-1 supplier knows the outcome of the L1 negotiation while the tier-2 firms, in general do not. We thus assume that the tier-2 firms have an unbiased estimate of the outcome, and that based on this estimate they behave the same as if they had precise information.
5. Results

5.1. The Hierarchical Core

The HC and the HSV will in general differ from the core and the SV, respectively, because some coalitions in \(2^M\) are excluded in the hierarchical structure. Chatain and Zemsky (2011) refer to such incomplete linkages as “frictions.” For the HC, excluded coalitions are the only source of potential differences to the core. The following lemma specifies under what conditions excluded coalitions exist. We relegate all proofs to the Appendix.

**Lemma 1.** A hierarchical game \(G_B\) is “restrictive” in the sense of excluding coalitions in \(2^M\) if and only if there exists a cluster \(M_i \in \mathfrak{B}\) with \(|M_i| \geq 2\) and \(|M \setminus M_i| \geq 2\).

For illustration, we consider a game with \(M = \{m_1, m_2, m_3, m_4\}\) and bargaining structure \(\mathfrak{B} = \{\{m_1, m_2\}, \{m_3, m_4\}\}\). This game fulfills the condition in Lemma 1, and indeed four coalitions are excluded (\(\{m_1, m_3\}\) etc.). For restrictive games, the HC differs from the core under quite general conditions:

**Proposition 1.** Let \(G\) be strictly convex and \(G_B\) restrictive. Then (a) the core of \(G\) is a proper subset of the HC of \(G_B\), i.e., \(C(M, v) \subset HC(M, \mathfrak{B}, v)\). (b) If a coalition \(K \subset M\) is excluded in \(G_B\), then the vanishing of the corresponding constraints on the core implies that \(HC(M, \mathfrak{B}, v) \setminus C(M, v)\) contains allocations in which \(K\) obtains less, as well as allocations in which \(K\) obtains more than in any allocation in \(C(M, v)\). (c) If \(G\) is convex then \(C(M, v) \subseteq HC(M, \mathfrak{B}, v)\).

Part (b) of Proposition 1 shows that, for strictly convex games, a hierarchical structure has ambiguous effects on the solution set when comparing the HC to the core. In our example, each core allocation \(x\) fulfills the condition \(x(\{m_1, m_3\}) \geq v(\{m_1, m_3\})\), while this restriction is absent for the HC. Due to strict convexity, this implies that the HC indeed contains allocations with \(x(\{m_1, m_3\}) < v(\{m_1, m_3\})\). However, also the restriction \(x(\{m_2, m_4\}) \geq v(\{m_2, m_4\})\) \(\iff\)
\( x(\{m_1, m_3\}) \leq v(M) - v(\{m_2, m_4\}) \) is absent, implying that the HC contains allocations with \( x(\{m_1, m_3\}) > v(M) - v(\{m_2, m_4\}) \). In general, there can be no dominance in the sense that, for given \( K \subset M \), all allocations in \( HC(M, \mathcal{B}, v)\setminus C(M, v) \) are superior or inferior for \( K \) to those in \( C(M, v) \). Thus, a statement as to whether the hierarchical game is advantageous for player \( m_i \) or not requires comparing those allocations in \( HC(M, \mathcal{B}, v)\setminus C(M, v) \) that are advantageous for \( m_i \) relative to its allocations in \( C(M, v) \) to those that are disadvantageous. Essentially, the difference between the core and the HC is that the latter contains distributions that are less “balanced” within clusters: if the coalition \( K \) is excluded in the hierarchical game and \( m_i \in K, m_i \in M_j \), then \( m_i \)’s allocation in the HC is not bounded from below by the constraint, \( x(K) \geq v(K) \), nor bounded from above by the constraint, \( x(M\setminus K) \leq v(M\setminus K) \). Part (c) of Proposition 1 shows that, if \( G \) is convex but not strictly convex, the core of \( G \) and the hierarchical core of \( G_{\mathcal{B}} \) may be identical.

For the L1 allocations in the HC we obtain a clear result. To formulate it, we define a mapping \( f: \mathbb{R}^n \rightarrow \mathbb{R}^k, x \mapsto f(x) = y \) such that \( y_i = x(M_i) \).

**PROPOSITION 2.** For convex games, \( f\left( C(M, v) \right) = HC_{L_1}(\mathcal{B}, \hat{v}) \).\(^6\)

Proposition 2 implies that, using the core and the hierarchical core as solution concepts, the move from a non-hierarchical to a hierarchical bargaining structure has no effect on the overall payoff of a group of firms that, in the hierarchical bargaining structure, form a cluster. In technical terms, the allocations among the \( k \) clusters that lie within the HC are the same as those that result from the core allocations of the linear game by summing up the individual players’ allocations within each subset that corresponds to a cluster. In the four-player example, an allocation is in the L1 HC if the cluster \( \{m_1, m_2\} \) receives at least \( v\{m_1, m_2\} \), the cluster \( \{m_3, m_4\} \) receives at least \( v\{m_3, m_4\} \), and both together receive \( v(M) \). The exact same conditions hold for the allocations that these coalitions receive in the core.
In the following, we thus focus on the SV when comparing hierarchical games to their linear counterparts, but will also address the core in Propositions 3 and 5. Since the analysis of a general hierarchical game is complex and not insightful, we study a number of simplified cases.

5.2. All Firms Essential

We first analyze the case that all firms are essential, with a general bargaining structure given by \( \mathcal{B} = \{M_1, ..., M_k\} \). From the definition of the HSV it follows that each cluster receives the same payoff, \( v(M)/n \), which is equally shared among the players within the cluster. In contrast, according to Proposition 1c the HC of \( G_\mathcal{B} \) is a proper or improper superset of the core of \( G \). Since the latter is maximal if all firms are essential, both are identical. We summarize these results in

**Proposition 3.** If all firms are essential and \( \mathcal{B} = \{M_1, ..., M_k\} \), then for firm \( i \) in cluster \( j \) the HSV of \( G_\mathcal{B} \) is greater than the SV of \( G \) if \( |M_j| < n/k \). The HC is identical to the core and comprises all allocations \( x \) that satisfy \( x_i \geq 0 \) and \( x(M) = v(M) \).

Thus according to the SV and the HSV, smaller clusters—with a size below the average—fare better in a hierarchical than in a linear bargaining structure if all firms are essential. This finding is intuitive and in line with the three-firm example presented in the Introduction. The core and the HC solution concepts, in contrast, indicate no differences between the bargaining structures, neither on L1 nor on L2.

5.3. Bargaining Structures with one Cluster and one Single Firm

We now address the case of general \( v \) and a specific bargaining structure given by \( \mathcal{B} = \{\{m_1, ..., m_{n-1}\}, \{m_n\}\} \). That is, all firms except \( m_n \) form a cluster, \( M_1 \). Using the definitions of the SV, the HSV, and complementarity gains introduced earlier, it is straightforward to calculate, for the single L1 firm \( (m_n) \), the difference between both as
\[
\phi_n^{HSV}(v) - \phi_n(v) = \frac{1}{2} \sum_{S \subseteq M_1, S \neq \emptyset} \frac{|S|! (n - |S| - 1)!}{n!} \left( \Delta_{M_1}(m_n) - \Delta_S(m_n) - \Delta_{M_1 \setminus S}(m_n) \right),
\] (11)

which for \( n = 3 \) simplifies to

\[
\phi_3^{HSV}(v) - \phi_3(v) = \frac{1}{6} \left( \Delta_{\{m_1,m_2\},\{m_3\}} - \Delta_{\{m_1\},\{m_3\}} - \Delta_{\{m_2\},\{m_3\}} \right).
\] (12)

The above terms suggest the notion of super-complementarity: \( v \) is super-complementary if the corresponding complementarity gains function, \( \Delta \) is complementary in each of its arguments.\(^7\) In a way, complementarity gains \( \Delta_{J,K} \) correspond to the second derivative of the characteristic function, and the expressions in (11) and (12), to the third.\(^8\) We thus obtain:

**Proposition 4.** Being the single L1 firm, \( m_n \), in a hierarchical \( n \)-firm bargaining structure with one cluster, \( M_1 \), is superior to being in a linear bargaining structure if and only if, in the weighted average over all \( S \subseteq M_1 \) as described by (11), the focal firm’s complementarity gains \( \Delta_{M_1}(m_n) \) exceed the sum of the complementarity gains from joining \( \{m_n\} \) with \( S \subseteq M_1 \) and with the complement of \( S \) in \( M_1 \) separately. A sufficient condition for this to be true is that the characteristic function exhibits super-complementarity.

Hence, the single L1 firm benefits from hierarchical bargaining if top-level complementarity is large. In turn, if most complementarity is realized between this firm and subsets of the cluster, then this position hurts \( m_n \)’s value capture compared with it being in a linear bargaining structure. For practical implications, consider an OEM that integrates all suppliers into one mega-supplier. Doing so is advantageous for the OEM if the value gain realized by combining the OEM’s contribution with that of the mega-supplier is large, and the value gains realized by combining it with the contributions of subsets of the suppliers are small.

To flesh out this result, we analyze a symmetric three-firm bargaining situation. For the characteristic function, we assume that one firm alone captures the value of \( \alpha \), two firms, \( \beta \), and
all firms together, 1. From convexity it follows that $\beta \geq 2\alpha$, $1 \geq \alpha + \beta$, and $1 + \alpha \geq 2\beta$. For a hierarchical bargaining structure to be weakly preferable for the single L1 firm to a linear one, equation (12) yields the condition, $1 - 3\beta + 3\alpha \geq 0$. Figure 1a illustrates this example. The shaded area represents all parameter combinations that are consistent with $v$ being convex. In the lower part of this area, a hierarchical structure is advantageous for the single firm on L1.

--- Insert Figure 1 about here ---

The intuition why small $\beta$ is advantageous for $m_3$ in a hierarchical structure is the following. A large value of $\beta$ means that, from the perspective of $m_3$, the other two firms show a substitutive rather than a complementary relationship: the additional value that both together bring for $m_3$, $1 - \alpha$, is less than the sum of the additional values that each brings individually, $2(\beta - \alpha)$. Intuitively, thus, when $\beta$ is large, then in the linear bargaining structure $m_3$ plays the other two firms off against each other, while in the hierarchical structure they form a cartel.

It is furthermore insightful to consider the case that the single firm is essential. In that case, due to convexity of $v$, the final term in (11) is non-negative for all $S$. As a result, negotiating with one cluster is always weakly preferable for $m_n$ compared to negotiating in a linear structure. However, the essential firm may capture an even higher share of the value as a member of a cluster of $n - 1$ players, as we now show for the case of three players. The characteristic function is given by $v(\{m_3\}) = \alpha$, $v(\{m_1, m_3\}) = v(\{m_2, m_3\}) = \beta$, and $v(\{m_1, m_2, m_3\}) = 1$. The shading in Figure 1b indicates parameter combinations that are consistent with convexity. Using equation (8) for the characteristic function of the game played within a cluster, we can distinguish three areas. For $\alpha > (3\beta^2 + 2\beta - 1)/4$, in Area I, the essential firm does best as the single player in a hierarchical bargaining structure, and worst as a member of the cluster. Its payoff in the linear structure lies in between these extremes. For $(\beta^2 + 2\beta - 1)/2 < \alpha < (3\beta^2 + 2\beta - 1)/4$, in Area
II, being the single L1 player in a hierarchical structure is still best for $m_3$, but as a member of the cluster is achieves a higher payoff than in a linear structure. Finally, for $\alpha < (\beta^2 + 2\beta - 1)/2$ (Area III) it is optimal for the essential firm to be a member of the cluster, while it does worst in linear bargaining. Thus, being part of the cluster can be advantageous for an essential firm if the complementarities realized within the cluster are relatively large compared to those realized on L1.

In Area IV, the assumption of convexity is not fulfilled, but the SV can be calculated and the core is non-empty. Extending the above analysis to this area shows that, as in the adjacent Area I, being a member of the cluster is best for the essential firm, but linear bargaining becomes preferable to negotiating with the cluster $\{m_1, m_2\}$. Overall, thus, the optimal bargaining structure for the essential firm is always a hierarchical one.

The core solution concept again yields identical results for both bargaining structures:

**PROPOSITION 5.** If $\mathcal{B}$ consists of a cluster $M_1$ and a one-element set $\{m_n\}$ then the HC of $G_{\mathcal{B}}$ is identical to the core of $G$.

5.4. Symmetric Game with two Clusters

We now turn to a general two-cluster structure, $\mathcal{B} = \{M_1, M_2\}$, while assuming for the sake of transparency symmetry of the characteristic function. That is, $\nu(J) = \nu_{|J|}$ for all $J \subseteq M$. For symmetry reasons, the SV is the same for all players and identical to $\nu_n/n$, and the HSV is identical for all players within the same cluster. For $m_i \in M_1$ and $|M_1| = k$ we obtain:

$$\phi_l^{HSV}(\nu) - \phi_l(\nu) = \frac{1}{2k}(v_k + v_n - v_{n-k}) - \frac{\nu_n}{n}$$  \hspace{1cm} (13)

We introduce for $1 < j < n$, $\delta_j := (j/n)v_n - v_j$, which measures how much $\nu$ deviates, at $j$, from the linear case—in other words, how much of the value contribution that $j$ players make to the grand coalition is not yet realized in a coalition of size $j$. With this definition, (13) becomes:
\[
\phi_i^{HSV}(v) - \phi_i(v) = \frac{1}{2k} (\delta_{n-k} - \delta_k).
\]  

(14)

This term is positive if the downward deviation \(\delta_k\) of the focal cluster \(M_1\) from the linear case is smaller than that of the complementing cluster, \(M_2\). Since the ‘missing deltas’ are realized on L1, they are shared equally between the clusters, therefore the cluster with the smaller \(\delta\) benefits.

The specific case of \(v(f) = |f|/n\) serves as an illustration. For \(z = 2\), (13) vanishes—an instance of a characteristic function that is convex, but not super-complementary. For \(z = 3\), which implies that \(v\) is super-complementary, (13) equals \((1 - k/n)(1 - 2k/n)/(2n)\), which is positive for \(k < n/2\). Thus, in a two-cluster structure the members of the smaller cluster fare better than in a non-hierarchical structure. In particular, hierarchy is advantageous for a single L1 firm facing a cluster of \((n - 1)\), as analyzed above.

We summarize these findings and generalize them to \(z > 1\) in the following proposition.

**Proposition 6.** In a symmetric game, a two-cluster hierarchical structure is advantageous, in terms of the HSV vs. the SV, for the cluster with \(k\) players if \(\delta_k \equiv (k/n)v_n - v_k\) is less than \(\delta_{n-k} \equiv ((n - k)/n)v_n - v_{n-k}\). For \(v(f) = |f|/n\), \(z > 1\), the smaller cluster benefits compared with a linear structure if \(z > 2\), while the larger cluster benefits if \(1 < z < 2\).

5.5. **Merging of Bargaining Positions**

A given supply chain and the bargaining structure that it entails may be changed by a merger of two constituent firms or clusters on L1. For example, when an OEM restructures its value chain a former tier-1 supplier may be moved to tier-2 and thus become part of a cluster. Since it is irrelevant for the L1 allocation if the parties are firms or clusters, we assume that there are \(n\) firms initially, two of which subsequently merge. We analyze the effect on the allocations of the remaining firms. A straightforward calculation shows:
PROPOSITION 7. With $\mathcal{B} = \{\{m_1\}, \ldots, \{m_{n-2}\}, M_1\}$ and $M_1 = \{m_{n-1}, m_n\}$, the difference between $m_1$’s HSV and its SV equals

$$\phi_1^{HSV}(v) - \phi_1(v) = \sum_{S \subseteq M \setminus\{m_{n-1}, m_n\}} \frac{(|S| + 1)! (n - |S| - 2)!}{n!}. \left(\Delta_S,\{m_1\} + \Delta_{S \cup M_1},\{m_1\} - \Delta_{S \cup \{m_{n-1}\},\{m_1\}} - \Delta_S,\{m_n\},\{m_1\}\right),$$

which, for the case of a symmetric characteristic function, with $v(J) \equiv v_{|J|}$, simplifies to

$$\phi_1^{HSV}(v) - \phi_1(v) = \sum_{s=0}^{n-3} \frac{(s + 1)(n - s - 2)}{n(n - 1)(n - 2)} (v_{s+3} - 3v_{s+2} + 3v_{s+1} - v_s). \quad (16)$$

A consolidation of two L1 parties is beneficial for a stand-alone firm $m_1$ if (15) is positive. This is particularly true if complementarity gains between $m_1$ and a coalition $S$ grow stronger than linearly when players are added to $S$—i.e., if $G$ exhibits super-complementarity. Equation (16) illustrates this point, where the final term is a discrete version of the third derivative of $v_s$.

An interesting case is that of $m_1$—the OEM in a value chain, say—being essential. A merger of $m_{n-1}, m_n$ is beneficial for $m_1$ if, in the weighted average over all subsets $S$ of players, the complementarity that the merging firms together add to the coalition of $m_1$ and $S$ is greater than the sum of what they add individually. This pattern is familiar from Section 5.3.

As an example, think of an essential OEM of tablet computers, and assume that the various connectivity technologies (LTE, WiFi, Bluetooth, USB) come from different suppliers. These technologies fulfill similar functions and therefore should not be super-complementary. Thus, our model predicts that, from a value capture perspective, it is preferable for the OEM to negotiate with each of the suppliers on L1, rather than bundle them into a cluster and negotiate with a cluster representative. In contrast, in negotiating with holders of standard essential patents on LTE, which by definition are strictly complementary and, thus, exhibit super-complementarity, the OEM benefits when all or some of these form clusters.
5.6. Rearranging Firms between Clusters

Changes to a value chain may also imply that the firms constituting the clusters are rearranged while the number of clusters remains constant. For example, the OEM may move a tier-2 supplier to a different tier-1 supplier, or may ‘break up’ a tier-1 supplier in the sense that one of its units becomes a tier-2 supplier to a different cluster. In the latter case, the firm would have multiple roles in a value chain and multiple positions in the bargaining structure (cf. Luo et al. 2012).

By direct calculation, we derive:

**Proposition 8.** With \( \mathcal{A} = \{\{m_1\}, ..., \{m_{n-2}\}, M_A\}, M_A = \{m_{n-1}, m_n\}, \mathcal{B} = \{\{m_1\}, ..., M_B, \{m_n\}\}, M_B = \{m_{n-2}, m_{n-1}\}, \) the difference between \( m_1 \)’s HSV in cases A and B equals

\[
\phi_1^{HSV, A}(v) - \phi_1^{HSV, B}(v) = \sum_{S \subseteq \{m_2, ..., m_{n-3}\}} \frac{|S| + 1}{(|S| - 3)!} \cdot \left( \Delta_{SU(m_{n-2}),m_1} + \Delta_{SU, m_1} - \Delta_{SU, m_1} - \Delta_{SU(m_{n})},m_1) \right),
\]

which for the case of \( n = 4 \), with \( \Delta_{i,j,k} := \Delta_{\{m_i\},\{m_j,m_k\}} \) etc., simplifies to:

\[
\phi_1^{HSV, A}(v) - \phi_1^{HSV, B}(v) = \frac{1}{6} \left( \Delta_{1,2} + \Delta_{1,34} - \Delta_{1,23} - \Delta_{1,4} \right).
\]

Thus, an OEM in a position to restructure its value chain should try to group those suppliers that jointly create large complementarity gains with the OEM itself into the same cluster, and leave those firms that individually create large complementarity gains with the OEM ungrouped.

Such reshuffling of positions in the value chain may have strong implications also for the firms subject to it. Assume the cluster \( M_A \) in Proposition 8 is one firm with two units, which after the reshuffle hold different positions in the bargaining structure. For simplicity, consider the case of \( n = 4 \) with symmetry, i.e., a coalition of 1/2/3/4 players can capture the value of \( \alpha/\beta/\gamma/1 \). One can show that having two positions in the bargaining structure, \( m_4 \) on the top level and \( m_3 \) as part of the cluster \( M_B \), is advantageous for the focal firm to having a single position, as the cluster \( M_A \).
if $\gamma - \alpha < 1/2$. The intuition behind this result is that large $\alpha$ implies that the value increase that $M_A$ makes by joining with a single firm is smaller, while small $\gamma$ means that large complementarity gains are realized in the final step, when all players form the grand coalition. Since these gains distribute equally over the negotiators involved, it is advantageous for the focal firm to be part of two of them. In particular, if all firms are essential and thus $\gamma = 0$, then $m_3$ and $m_4$ together would obtain $1/3$ as parts of the cluster $M_A$, and $1/2$ when split as in the bargaining structure $\mathcal{B}_B$.

6. Drivers of Bargaining Structure and Predictions of the Model

Our model analysis has shown that bargaining structure should matter for value capture. However, if technical boundary conditions and other factors exogenous to strategic management completely determined bargaining structure, the model would serve only to predict the effects of technical or environmental changes. We now show that this is not the case. Rather, bargaining structure is malleable and mostly follows value chain architecture, which in turn, is to a good extent under the control of the central firm (Jacobides et al., 2015; Novak and Wernerfelt, 2012). Thus, our analysis can provide guidance for managerial decision makers. We present examples of firms that actively shape the bargaining structure of their value chains, identify the levers they apply, demonstrate to what extent value chain architecture mirrors bargaining structure, and assess if the firms’ actions are consistent with the hierarchy strategy.

6.1. Method

Since the effect of product architecture and value chain architecture on bargaining structure has not yet received research attention, we choose a qualitative approach. From a long list of potential cases from different industries developed with the help of experts, practitioners, and literature, we selected two contrasting cases to facilitate the identification of general patterns (Eisenhardt, 1989):

1. home appliances, in particular the T20 laundry dryer by Bosch Siemens Hausgeräte (BSH),
and (2) large, commercial aircraft, in particular the Airbus A350 and the Boeing 787 Dreamliner. These cases strongly differ; while long product cycles, low-volume products, high technological requirements and a high degree of specialization of players characterizes the aircraft industry, the home appliances market is a mass market with more or less standardized products.

We base our case studies on (a) interviews with practitioners, (b) publications, and (c) databases. In total, we conducted semi-structured interviews with 10 managers who played an active role during the design and production of the focal products (see Table 1). The interviewees cover firms from different levels of the value chain, ranging from OEM to tier-2 suppliers, hence, giving a clear view on value chain and bargaining structure. All interviewees have extensive experience with supplier interaction.

--- Insert Table 1 about here ---

A broad set of secondary sources enriches our collected data. This includes internal documents of BSH, publications on the T20 dryer and aircraft programs, and the Airframer database covering more than 4,000 aircraft suppliers.

6.2. Modularization of Laundry Dryers at BSH

BSH is one of the largest producers of home appliances selling laundry dryers and other products to consumers. In 2005, the laundry dryer business unit initiated a project to counter the increased complexity resulting from a larger variety of available products and the need to reduce costs. At the same time, there was a push to focus on BSH’s core competencies, which, in the business segment of laundry products are cleaning and drying.

The answer to these issues was the new, modular laundry dryer T20. It consisted of six systems that could be produced independently: base, front, back board, drum, control panel, and door module. The change in product architecture went along with a change in the value chain
architecture. With the design of independent systems, BSH introduced system suppliers to taking over responsibility for the development and production of these parts. In particular, these suppliers performed the integration of the systems, a task mostly done previously by BSH; only the back board, the drum, and the final integration remained with BSH. Through the establishment of system suppliers, a number of firms moved from tier-1 to tier-2 in the value chain.

Turning to bargaining structure, we observed that for most parts it changed in parallel to the value chain architecture. For example, as E.G.O. Blanc und Fischer & Co. (E.G.O.) became the system supplier for the control panel, it took over responsibility for managing the tier-2 suppliers, including Prettl, which delivered plastic parts, tooling equipment, and wiring harnesses for the control panel, and was once a direct supplier to BSH. For the T20, Prettl negotiated prices and volumes with E.G.O. with almost no interaction with BSH. Thus, the change in product architecture that BSH had performed entailed corresponding changes in both the value chain architecture and in the bargaining structure.

While the bargaining structure paralleled the value chain architecture in most instances, we also observed deviations. BSH decided to keep responsibility for the procurement of a few selected tier-2 components, which had one of the following characteristics: BSH could realize a higher volume effect by purchasing them directly, or the components were crucial for differentiating features of the product. While system suppliers still managed the supply of these components and even negotiated contract details such as terms of payment, delivery date, and financing, price negotiations remained with BSH. For example, BSH directly procured the specific granulate that creates the characteristic white color associated with BSH home appliances and then provided it to system suppliers. Similarly, Arc International, the supplier of window glass for the door complex, directly negotiated prices with BSH. In that case, however, Arc International sought the position of direct negotiations with the OEM.
BSH and Arc International share a common trait; both have strong positions in the value chain and are nearly non-replaceable. BSH, in its role as the OEM, had some unique assets and capabilities such as its brands and customer access, and explicit and tacit knowledge of product design and production process. Furthermore, its system suppliers were dependent on BSH as it represented a significant share of their revenues. Similarly, Arc International was in a dominant position as the only producer able to offer the window glass for a competitive price. Further, the laundry dryer business represented only a minor part of its overall business, which increased Arc International’s bargaining power. These findings suggest that BSH and Arc International’s strong positions allowed them to shape the bargaining structure, in such a way that it deviated from the value chain architecture.

With regard to value capture, the results from our model analysis suggest that BSH benefitted from the consolidation of tier-1 suppliers and the new hierarchical bargaining structure. The situation can be modeled as a bargaining structure with two essential firms, BSH and Arc International. After the change, BSH bargained with a smaller number of relatively non-essential direct system suppliers instead of a large number of component suppliers. As we can assume that complementarity is largest in the final integration step (as BSH’s assets come into play), our model predicts that the new bargaining structure had a positive effect on BSH’s value capture. Interestingly, we have no indication that explicit considerations of bargaining played any role for BSH in its move to a new product and value chain architecture.

On the supplier side, there are winners and losers of the change. First, the model indicates that Arc International could benefit from the increased hierarchy through its position in top-level negotiations. Its push to keep that position and directly negotiate with BSH is in line with the hierarchy strategy. Further, for firms that were moved to a lower level in the bargaining structure, the change was likely disadvantageous. Finally, whether suppliers that became system suppliers
could benefit from the new bargaining structure mainly depends on their ability to decrease replaceability. If they cannot gain a stronger position, the new bargaining structure is most likely disadvantageous to them; however, their role as integrators may allow them to decrease replaceability over time by deepening the relationship to the OME, thus gaining a relation-based competitive advantage (Dyer and Singh, 1998).

6.3. System Suppliers in Commercial Aircraft Manufacturing

The last decade has seen significant structural changes in aircraft value chains. Both Airbus and Boeing introduced an additional level to their tiered supplier structure, with the aim to reduce complexity and administrative cost and to spread the risks of development and production. Large system suppliers, so-called mega suppliers, have taken over production of major sections of the aircraft and supply fully integrated parts, for example, the nose section or the wings, to the OEMs. Prominent recent examples are the Airbus A350 and the Boeing 787 Dreamliner programs.

The new tier-1 suppliers not only deliver integrated sections to the OEM but also manage the supply chain for their respective system; thus, supplier relations once with the OEM, are now in the hands of the system supplier. For example, Boeing empowered 12 selected suppliers to have control and ownership of the independently built large modules of the 787.

Mega suppliers have full responsibility for the value chain of their respective sections of the aircraft. Within boundaries defined by the OEMs to ensure technical capabilities and mitigate risks, they have the freedom to select lower tier suppliers and run negotiations with them independently. The changes in the value chain architecture introduced an additional layer, making the bargaining structure more hierarchical. During one of our interviews, a former senior vice president of procurement strategy at Airbus stated: “We negotiate at system level, not component level.”
For example, for the Airbus A350, Spirit AeroSystems has taken over the design and production of the center fuselage section from Airbus. While Airbus once directly purchased components and subsystems for other aircraft programs to install in the fuselage—such as the oxygen system—Spirit AeroSystems now manages and directly negotiates with these suppliers for the A350 program.

However, as for BSH there are instances in which value chain architecture and bargaining structure do not coincide. For certain commodities—in particular raw materials such as titanium, aluminum-lithium, and carbon fiber—OEMs negotiate enablement contracts with the respective tier-4 suppliers to ensure competitiveness through a low guaranteed price and secure the required capacity. Suppliers are free to tap into the volume negotiated by the OEM, but may use other sources as well. Likewise, OEMs seek to control a few selected critical suppliers to mitigate technical risks and costs. In contrast to the case of BSH, apart from minor changes related to interfaces (which were consequences rather than drivers of the changes to the value chain), product architecture played only a minor role in our aircraft cases.

Beyond the OEMs, a large number of firms in the aircraft value chain can be considered as nearly essential for three reasons. First, a few specialist firms dominate many component and system markets because of high technical complexity, administrative requirements such as certifications, and logistic requirements. Second, switching costs are high due to interdependency of systems, which drives complexity, risk, and cost involved in switching suppliers. After the start of an aircraft program, design changes requiring a switch of suppliers would delay the program for months. Third, airlines appreciate consistency in the supplier base. Having different suppliers for the same component adds to maintenance costs and complexity for airlines. All these aspects put suppliers in a rather strong position, in particular when additional negotiations during the design and ramp-up phase are required due to new, upcoming requests from the OEM. The fact that many
suppliers are nearly essential implies that complementarity in most negotiations is high, since only a complete subsystem is of value to the next level in the value chain.

The results from Sections 5 indicate that the new value chain architecture and bargaining structure should have a positive effect on the value capture of OEMs. While many nearly essential firms are moved to tier-2, the OEMs keep their top-level positions in the new hierarchical bargaining structure. For example, B/E Aerospace supplies oxygen systems to both the Airbus A350 and the Boeing 787. The oxygen system is a highly specialized component with only two capable suppliers on the market. In the past, B/E Aerospace was a tier-1 supplier that negotiated directly with the OEMs. In contrast, for the most recent designs B/E Aerospace supplies to and negotiates with tier-1 suppliers. According to our model results, the new position in the bargaining structures hurts the value capture of B/E Aerospace, while it benefits Boeing and Airbus. This finding is particularly interesting since it is counterintuitive; one might assume that the OEMs, owing to their sheer size, are more powerful counterparts in negotiations than any tier-1 supplier. However, size by itself is not a driver of bargaining power, and all other negotiation advantages of the OEM—in particular purchasing volume and low replaceability—are passed on to the tier-1 supplier that, in a way, negotiates on the OEM’s behalf.

Further, in the old structure, having several inputs that were negotiated at the top-level with the OEM did not provide additional benefits as bargaining positions could not be kept separate. Through the introduction of the more hierarchical bargaining structure, a firm providing several inputs may find itself supplying to more than one mega supplier. As a result, it occupies several separate bargaining positions and thus benefits with regard to value capture.

As for BSH, our interviews provide no indication that bargaining aspects influenced the aircraft makers when devising the new value chain architectures. This ignorance may be risky. The above example of firms that, after the change, occupy several separate bargaining positions
illustrates this point. Thus, consolidation and a more hierarchical value chain are not beneficial to the OEM in all aspects.

7. Discussion

Different value chain architectures may yield the same final product. This variation raises the questions of if and how a firm’s position in a hierarchical value chain affects the value it can capture, other things being equal. We address this question using cooperative game theory, in particular the concepts of bargaining structure, the hierarchical Shapley value, and the hierarchical core. A qualitative empirical study complements our model analysis.

7.1. Summary of Results

Our central finding regarding the core solution concept is a non-result. We find that the hierarchical core places the same bounds on the amount of value that a cluster can capture in a hierarchical bargaining structure as the core does in a linear bargaining structure for the group of players that form the cluster. In particular, the bounds for the value capture of a single firm do not change when the other negotiators form clusters. Since the core is silent on the relative probability of the various solutions that it contains, this result does not mean, however, that the introduction of hierarchical bargaining does not have an effect on the resulting distribution.

The Shapley value yields results that are more concrete. For stating them, the concepts of complementarity gains and of super-complementarity prove useful. The former refers to the incremental value that two coalitions can capture jointly over the sum of what each can achieve individually. Super-complementarity, in turn, means that the complementarity gains between a given coalition and some other coalition $S$ are super-additive in $S$. Intuitively, this means that larger complementarity gains are realized toward the final levels of the value chain.
If the non-hierarchical game exhibits super-complementarity, then, with some simplification, a participant in the top-level negotiation benefits if other participants are merged into clusters. This finding resembles the familiar game-theoretic result that, with a number of producers of perfectly complementary goods, a producer benefits when other producers merge. However, our results are far more general.

In more detail, our main results are the following. If the underlying game exhibits super-complementarity, (i) a bargaining structure consisting of one firm and one cluster is advantageous to the single firm compared with linear bargaining, (ii) a two-cluster hierarchical structure in a symmetric game is advantageous to the smaller cluster, and (iii) a merger of two firms or clusters on L1 is advantageous for the other L1 negotiators. If all firms are essential—the extreme case of super-complementarity—then members of smaller clusters fare better in a hierarchical than in a linear negotiation structure.

Our qualitative empirical study yields the following complementing findings. Value chain architecture is malleable, and to some extent under the control of the central firm. Modular product architecture can be leveraged to influence the value chain architecture. Bargaining structure mostly follows value chain architecture, but powerful players may establish exceptions. Managers do not consciously consider value capture aspects when devising a bargaining structure.

7.2. Link to the Literature

Our findings suggest that bargaining structure and the underlying value chain architecture can be a source of competitive advantage, provided they are stable or under the control of the focal firm. Our research thus relates to the literature on value capture theory, recently reviewed by Gans and Ryall (2017). Value capture theory is concerned with variations in competitive intensity along the value chain and the factors that cause it, and like our study, applies cooperative game theory. The challenge in linking our results to this literature lies in the fact that the latter typically employs the
core solution concept, which in the setting we study predicts no effect of bargaining structure on the competitive intensity between the clusters. The Shapley concept, in contrast, does make concrete predictions, but is typically eschewed by value capture theory scholars because of the strong assumptions it makes. A way to reconcile these views may be to interpret the determinants of the Shapley value, i.e., the marginal contributions that a player makes to the various coalitions, as persuasive resources, which help a player capture more value than its competitive minimum (Gans and Ryall, 2017; Montez et al., 2017).

A contribution to the literature on value capture theory particularly closely related to our work is the study by Chatain and Zemsky (2011). The authors analyze the effect of frictions on value creation and value capture that arise from incomplete linkages in the industry value chain. That is, a buyer cannot negotiate with all potential sellers, nor can a seller with all potential buyers. This assumption is similar to ours that firms in a hierarchical bargaining structure negotiate only with other firms on the same level and in the same cluster, and have limited information transparency across clusters. The main difference is that Chatain and Zemsky (2011) focus on frictions as the source of imperfect competition between substitutive firms, while our model emphasizes the effect of missing linkages when firms are complementary to each other. Common to their study and ours is the insight that incomplete linkages may support resource-based competitive advantages.

Our results also fit with the relational view of the firm, which holds that complementary resources and capabilities can generate interorganizational competitive advantage (Dyer and Singh, 1998). In a similar fashion, a firm tends to benefit from a particular bargaining structure if it positions itself in negotiation rounds that realize large complementarity gains. We extend the relational view insofar as we consider variations not in the set of firms that collaborate, but in how a given set of firms are linked among each other by the bargaining structure.
The theme of complementarity is central also to the studies by Adegbesan (2009) and Wernerfelt (2011). The authors argue that buyers on strategic factor markets are typically heterogeneous with respect to complementarities between their resources and those they seek to acquire. As a result, firms with greater resource complementarities to a given factor are likely to gain from trade in this factor. These findings parallel ours insofar as being part of a negotiation round is akin to acquiring the resources that the other firms in this round provide.

Furthermore, our results relate to the literature on industry architecture. Innovators designing the overall modular product architecture of a new product have the opportunity to shape, within boundaries given by technical limitations and the capabilities of suppliers, the value chain architecture (Baldwin and Clark, 2000; Colfer and Baldwin, 2010; Henderson and Clark, 1990; Jacobides et al., 2015; Novak and Wernerfelt, 2012). Value chain architecture, in turn, appears to be the main determinant of the bargaining structure. In particular, decisions on the top-level contributors in a value chain determine the bargaining situation of the system designer, who can exercise this power to enhance its value capture. Since a bargaining structure defines a hierarchy of negotiations, we denote a firm’s approach to optimizing its value capture through creating a favorable bargaining structure in its value chain as the hierarchy strategy.

The normative prescriptions of our model differ from those made by other applicable theories. Transaction cost economics (Williamson, 1979) recommends minimizing the risk of opportunistic behavior, which, in general, is unrelated to the main driver of our results, that is, complementarities between actors. Cognitive mechanisms as analyzed by Bardolet et al. (2011), in turn, would work toward a more even distribution of payoffs within one negotiation round; thus, reducing the number of negotiators it faces should always be desirable for a firm that has the power to do so (e.g., the OEM in the top level negotiation). In contrast, our model predicts that consolidating negotiators is advantageous only when complementarities between the focal firm
and the newly formed cluster are strong and disadvantageous otherwise. Finally, a focus on maximizing efficiency of knowledge flows (Lipparini et al., 2014) or on interdependencies between tasks (Novak and Wernerfelt, 2012) will yield other recommendations than our analysis for designing a value chain, unless knowledge flows between the firms involved, or relations between tasks, correlate in a suitable way with complementarities in joint value capture.

7.3. Implications for Managers

Implications for managers are twofold. First, our results suggest that managers take into consideration the resulting bargaining structure when devising product and value chain architectures. In some cases, a powerful firm may leverage its position to shape the bargaining structure to its advantage. Interestingly, most managers do not seem to be aware of how value chain architecture affects bargaining structure, and how the latter, in turn, affects value capture. Second, to achieve the above necessitates the involvement of various organizational functions besides the technical department, such as strategy and procurement, during the design of the product architecture to identify product and value chain architectures that involve beneficial bargaining structures. According to what we learned from practitioners, however, this is typically not the case. Rather, engineering devises the product architecture based on which procurement engages in negotiations with suppliers.

7.4. Limitations

There are several limitations to our analysis. To start with, the use of the SV and its generalization, the HSV, may come into question: The goal of the SV is to yield an allocation that is in some sense fair, but not necessarily the outcome of unrestricted bargaining. Nonetheless, it is more than a normative concept; empirically, researchers have found it to have a relatively good predictive accuracy (Michener et al., 1983; Michener et al., 1987). Furthermore, we do not claim that the point predictions that we derive for the SV and the HSV are correct. Rather, we argue that the sign
of the difference of a player’s SV or HSV between two bargaining structures is informative, and
provide a plausible economic interpretation of the effects we observe. The study by Bardolet et al.
(2011) on firm-internal budget allocation decisions between divisions supports the role of
clustering in the distribution of value. That said, research is needed to determine how well the
HSV predicts value splits in actual negotiations; an empirical test of the HSV could serve to assess
its predictive power. Additionally, the assumption of full transparency within, and zero
transparency across, negotiation rounds may raise questions. However, while these extreme levels
of transparency are probably not realistic, we maintain that transparency will be higher within than
across negotiation rounds.

Our model also does not account for the fact that value chain architecture and bargaining
structure may affect the overall value created. For example, splitting one essential position into
several to increase own value capture will hurt value creation not only because of increased
transaction costs but also because independently sold complements are priced excessively
(Cournot, 1897). Patent royalty stacking reflects this phenomenon (Shapiro, 2001) and is an
instance of the tragedy of the anticommons (Heller and Eisenberg, 1998).

Possible further extensions of the model could endogenize the emergence of the value chain,
regarding both its participants—which determine the overall value generated and distributed—and
its structure. Taking the value as a non-constant, which one could potentially model as a biform
game as proposed by Brandenburger and Stuart (2007), might provide an interesting perspective
on how the anticipation of eventual bargaining structures could matter during establishment of the
value chain. Similarly, the cost of changing bargaining structures is not a factor in the model.
Particularly in settings of existing buyer-supplier relationships, altering the bargaining structure
might raise costs and negatively affect the relations between firms.
The question also arises as to why firms that stand to lose from a particular bargaining structure would nonetheless accept it. We argue that firms involved only in lower-level negotiations do not have full transparency of the overall bargaining structure, and that only a few firms have the power to influence that structure. However, even for those that do command such power, there must be limits to the execution of the hierarchy strategy in order to sustain a healthy industry. In particular, the value capture of every firm needs to match at least the outside options beyond the focal value chain.

Finally, our empirical study complements the model analysis in various ways, but falls short of providing empirical evidence of the effects of value chain architecture on value capture. The challenge here is to find a setting in which these effects can be disentangled from concurrent ones, in particular, those of introducing a new product along with the new value chain architecture.

### 7.5. Conclusion

As the economy is increasing in complexity, the question of value chain architecture comes to the forefront. Our analysis has shown that, beyond a firm’s replaceability, the value chain architecture and the structure of negotiations that it codetermines affect its value capture. Especially in the early stages of an industry, as currently in the electrical car industry, value chain architectures and bargaining structures are in flux and subject to managerial action. With this paper, we contribute to the discussion of industry and value chain architectures, and the resulting linkages between firms, as driving forces of value appropriation.

### References


Hendrikse, G., 2011, "Pooling, access, and countervailing power in channel governance,"

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Figures and Tables

**Figure 1:** Hierarchical vs. linear bargaining, three firms; (a) symmetric, (b) one firm essential

![Hierarchical vs. linear bargaining, three firms](image)

**Table 1:** List of interviews

<table>
<thead>
<tr>
<th>Case</th>
<th>Company</th>
<th>Position</th>
<th>Role of interviewee</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dryer</td>
<td>BSH</td>
<td>OEM</td>
<td>Project Director T20</td>
<td>101 min*</td>
</tr>
<tr>
<td>Dryer</td>
<td>BSH</td>
<td>OEM</td>
<td>Vice President Purchasing</td>
<td>62 min</td>
</tr>
<tr>
<td>Dryer</td>
<td>Coko-Werk</td>
<td>Tier-1 supplier</td>
<td>Responsible sales manager and project manager T20</td>
<td>29 min</td>
</tr>
<tr>
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<td>Tier-1 supplier</td>
<td>Factory manager</td>
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</tr>
<tr>
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<td>Tier-2 supplier</td>
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<td>47 min</td>
</tr>
<tr>
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<td>Airbus</td>
<td>OEM</td>
<td>Head of strategic procurement for metal components</td>
<td>51 min</td>
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<td>OEM</td>
<td>Senior VP Procurement Strategy</td>
<td>39 min</td>
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<td>OEM</td>
<td>Executive VP</td>
<td>66 min</td>
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<td>OEM</td>
<td>Vice President Engineering</td>
<td>35 min</td>
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<td>Aircraft</td>
<td>HITCO Carbon Composites / Boeing</td>
<td>Tier-2 supplier / OEM</td>
<td>Executive VP / Senior Contract Manager and Senior Business Mgr.</td>
<td>53 min</td>
</tr>
</tbody>
</table>

Interviews were conducted between July and October 2014.

* Two separate interviews of 70 min and 31 min duration, respectively
Appendix

PROOF OF LEMMA 1. Assume a cluster $M_i$ as described exists in $\mathcal{B}$. Choose two players $m_j$, $m_k$ such that $m_j \in M_i$, $m_k \in M \setminus M_i$. Then in the solution concepts to $G_{\mathcal{B}}$ the coalition $K := \{m_j, m_k\}$ is excluded both on L1 (since only complete clusters are considered on L1) and on L2 (since only those coalitions are taken into account whose complement lies entirely within one cluster, while $M \setminus K$ has elements of both $M_i$ and $M \setminus M_i$). For the opposite direction, assume no cluster $M_i$ as described exists in $\mathcal{B}$. Then either $\mathcal{B}$ contains exclusively sets with exactly one element, in which case $G_{\mathcal{B}}$ is not different from $G$ and thus no coalition feasible in $G$ is excluded in $G_{\mathcal{B}}$; or $\mathcal{B}$ has exactly two elements, the cluster $M_i$ and a set containing exactly one player, $\{m_k\}$. In the latter case, again all coalitions feasible in $G$ feature in the characteristic functions (8) and (9) of $G_{\mathcal{B}}$; in particular, the coalitions in $2^M$ that contain $\{m_k\}$ and a subset of $M_i$ are captured by the terms $v(J \cup (M \setminus M_i))$. Thus, from the assumption that no cluster $M_i$ as described exists in $\mathcal{B}$ it follows that $G$ is not restrictive. Q.E.D.

PROOF OF PROPOSITION 1. (a) From strict convexity if follows that the core configuration is strictly complete, in which case the core is full-dimensional with $2^n - 2$ polyhedral faces of dimension $n - 2$ (Shapley, 1971). Each of these faces corresponds to one of the constraints defining the core. Some of these constraints vanish for the HC if $G_{\mathcal{B}}$ is restrictive, making the HC of $G_{\mathcal{B}}$ a proper superset of the core of $G$. (b) We first note that, if a coalition $K$ is excluded in $G_{\mathcal{B}}$, then its complement $M \setminus K$ is also excluded: to be excluded $K$ must contain a strict, non-empty subset of a cluster $M_i$ as well as a strict, non-empty subset of $M \setminus M_i$, in which case also $M \setminus K$ contains a strict, non-empty subset of $M_i$ (namely, $M_i \setminus (M_i \cap K)$) as well as a strict, non-empty subset of $M \setminus M_i$. 

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(namely, \((M \setminus M_i) \setminus K\)); thus, also \(M \setminus K\) is excluded in \(G_B\). Now consider the \(n - 1\) dimensional subspace of \(\mathbb{R}^n\) defined by the payoff of the grand coalition, \(x(M) = v(M)\). In this space, the subspaces of dimension \(n - 2\) defined by \(x(K) = v(K)\) and \(x(M \setminus K) = v(M \setminus K)\) are parallel to each other since the latter equation is equivalent to \(x(K) = v(M) - v(M \setminus K)\). They constitute opposite, \(n - 2\) dimensional boundary faces of the core of \(G\). Removing the constraint \(x(K) \geq v(K)\) \([x(K) \leq v(M) - v(M \setminus K)\] adds allocations to the HC that award less \([more]\) to \(K\) than any core allocation. (c) Follows from the fact that the conditions defining \(HC(M, B, v)\) are a subset of the conditions defining \(C(M, v)\). Q.E.D.

PROOF OF PROPOSITION 2.\(^1\) By Proposition 1c, \(C(M, v) \subseteq HC(M, B, v)\). Since, in general, the image \(\vartheta(F)\) of a subset \(F \subseteq H\) under a mapping \(\vartheta\) is a subset of the image \(\vartheta(H)\) of \(H\) under the mapping (i.e., \(\vartheta(F) \subseteq \vartheta(H)\)), it follows from the above that \(f(C(M, v)) \subseteq f(HC(M, B, v))\), which is identical to \(HC_{L_1}(B, \hat{v})\). For the opposite direction, we note that since \(f(C(M, v))\) is a convex set and \(f\) is linear it is sufficient to show that the extreme points of \(HC_{L_1}(B, \hat{v})\) are in \(f(C(M, v))\).\(^2\) Since the game \(G_B\) is convex, any extreme point \(y \in HC_{L_1}(B, \hat{v})\) is obtained from the increments of the characteristic functions when the clusters are ordered by some ordering, \(\omega\). That is, \(y_i = v\left(\bigcup_{l: \omega(l) \leq i} M_l\right) - v\left(\bigcup_{l: \omega(l) < i} M_l\right)\) (Shapley 1971, Theorem 3), where \(\omega\) is a bijective mapping of the set of clusters, \(B\), onto \(\{1, \ldots, k\}\). We extend the ordering \(\omega\) to an ordering \(\hat{\omega}\) of the player set, \(M\), in such a way that if \(m_j \in M_l\), \(m_{j'} \in M_{l'}\), and \(\omega(i) < \omega(i')\), then \(\omega(m_i) < \omega(m_{i'})\).

\(^1\) We owe this proof to Ron Perez.

\(^2\) Extreme points of a convex set are those that do not lie on any open line segment connecting two points of the set. Intuitively, the extreme points of \(HC_{L_1}(B, \hat{v})\) are its corners.
\( \omega(m_i') \). Define \( x \in \mathcal{C}(M, v) \) by \( x_j = v\left( \bigcup_{l: \hat{\omega}(l) \leq j} \{m_l\} \right) - v\left( \bigcup_{l: \hat{\omega}(l) < j} \{m_l\} \right) \). Since \( x(M_i) \) is a telescopic sum that equals \( y_i \), we have \( f(x) = y \). \textit{Q.E.D.}

**Proof of Proposition 5.** The HC of \( G_\mathcal{B} \) can differ from the core of \( G \) only if \( G_\mathcal{B} \) is restrictive, since the core as well as the HC are defined solely by the constraints regarding coalition payoffs. By Lemma 1, \( \mathcal{B} = \{M_1, \{m\} \} \) is not restrictive. \textit{Q.E.D.}

**Proof of Proposition 6.** With \( y := k/n \), the sign and the roots of (13) and (14) are determined by the term, \( ([1 - 2y] - [(1 - y)^z - y^z]) \). For \( y = 0, y = 0.5, \) and \( y = 1 \) this term vanishes. For \( 0 < y < 0.5 \), it is strictly concave if \( 1 < z < 2 \), and strictly convex if \( z > 2 \). Thus, it must be positive for \( 1 < z < 2 \) and negative for \( z > 2 \). The corresponding statement for \( 0.5 < y < 1 \) follows from point symmetry of the above term around \((0.5, 0)\). \textit{Q.E.D.}
This logic can be observed in the licensing of patents essential to communication standards such as LTE. By definition, each subset of a portfolio of standard-essential patents is itself essential, which explains why owners of small portfolios stipulate royalties far out of proportion to the size of their portfolios (Stasik, 2010).

This definition is in line with the notion of bargaining structure used, less formally, in the literature on interpersonal bargaining dating back to Krauss (1966). It was mostly examined in the context of union and wage bargaining as, for example, in Calmfors et al. (1988) and Moene et al. (1993).

The SV is the only solution concept for cooperative games characterized by efficiency (the full value is distributed), symmetry (players having the same value added to any given coalition receive the same value), additivity (combining two games yields a new game described by the sum of the two original characteristic functions), and invariance under dummy players (players without value added capture no value) (Shapley, 1953).

Similarly, Owen (1977) and Pulido and Sánchez-Soriano (2009) use different L2 characteristic functions in the definition of the Owen Value and the Coalitional Core, respectively. The functions they use are consistent with the respective underlying solution concept. In particular, the L2 characteristic function for the Owen Value is defined using the Shapley value.

The HC deviates in this regard from the coalitional core (Pulido and Sánchez-Soriano, 2009), in which $J \subset M_i$ can form a coalition with any set $S, S \subseteq \mathcal{B}\setminus\{M_i\}$, of clusters. In an analogous fashion, the HSV differs from the Owen value (1977).

We owe the proof to this proposition to Ron Perez.

Note that super-complementarity does not follow from convexity. For example, in the symmetric, convex game with $n = 3$ and $v(\{m_i\}) = 0$, $v(\{m_i, m_j\}) = 0.5$ $(i \neq j)$, and $v(\{m_1, m_2, m_3\}) = 1$, equation (12) yields $-1/12$.

In line with this interpretation, in the simple case of $v(J) = |J|^z$, convexity is given if $z \geq 1$ (implying that the second derivative of $v$ is non-negative, $v'' \geq 0$), while (11) is positive if $z \geq 2$ (implying $v''' \geq 0$).

BSH was a joint venture between Robert Bosch GmbH and Siemens AG until 2015, when Siemens sold its shares to Bosch. The firm’s name was then changed to BSH Hausgeräte GmbH.